"THE 35 GAME"

A 3-Player Dice Game for Introducing Inequality Models

Rosenthal Prize Submission

Robert Lochel

Hatboro-Horsham School District

Horsham, Pennsylvania

PURPOSE: The 35 Game is a dice-based game played with 3 students. During the gameplay, the teacher leads a class discussion about the possible outcomes of the game. Through the discussion students will encounter inequalities in three representations: verbal models, number-line models and, eventually, formal algebraic models. The game and ensuing discussion encourages students to move fluently through the three representations, discuss alternate models and develop a need for compound inequalities.

MATERIALS: 3 dice for each team. Student scoreboard for each student. Number line sheet for each student.

CONTENTS:

THE 35 GAME – COMMON CORE MATH STANDARDS, GRADE 6

Reason about and solve one-variable equations and inequalities.

CCSS.MATH.CONTENT.6.EE.B.5
Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

CCSS.MATH.CONTENT.6.EE.B.8
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < \mathrm{c}$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

THE 35 GAME – COMMON CORE MATH STANDARDS, GRADE 7

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

CCSS.MATH.CONTENT.7.EE.B.4
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

CCSS.MATH.CONTENT.7.EE.B.4.B
Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

THE 35 GAME – 3 STUDENTS, 3 DICE, 3 ROUNDS – GAME RULES

GOAL: The winner of this game is the student who comes closest to a total score of 35, without going over, after 3 rounds.

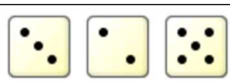
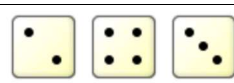
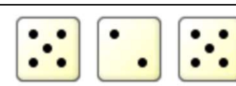
PROCEDURE: The three student players should establish who will roll first, second and third. Each student then rolls 3 dice and finds the sum of the top faces, which is recorded as their score for the round. After each student has completed their turn in round 1, round 2 begins and each sum is recorded. Students should then compute and record their total score after round 2.

In round 3, a student has the option to "pass" their turn and not roll in round 3. This option could be used if a player feels they are at risk of going over a total of 35 or if the player is confident that they will win the game without further rolls. Players who roll the dice in round 3 record their sum and find their updated total score. Note: most of the time, players in round 3 will choose to roll the dice.

At the end of 3 rounds, the player closest to a total score of 35 without going over wins the game. In the case of a tie, the teacher can either award co-winners or play a new game.

EXAMPLE GAME: The 3 students in this game have chosen to play in the following order: Julie, Colton, Aidan.

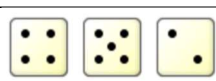
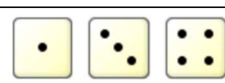
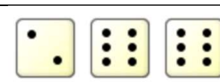
The rolls for round 1 are shown below, and the sums recorded in the student scoreboard:

Julie	Colton	Aidan
		

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →	Julie	Colton	Aidan
ROUND 1	10	9	12
ROUND 2			
TOTAL SCORE			
ROUND 3			
FINAL SCORE			

Round 2 rolls are then completed, with dice sums recorded and total scores after round 2 computed:

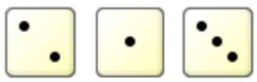
Julie	Colton	Aidan
		

Robert Lochel, Hatboro-Horsham SD

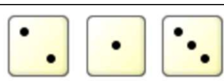
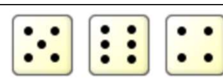
THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →	Julie	Colton	Aidan
ROUND 1	10	9	12
ROUND 2	11	8	14
TOTAL SCORE	21	17	26
ROUND 3			
FINAL SCORE			

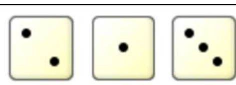
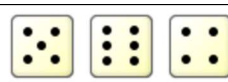
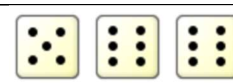
In round 3 players have the option to not roll the dice. Julie, who is behind, chooses to roll:

Julie	Colton	Aidan
		

Julie's new total of 27 puts her in the lead. Colton must now roll the dice:

Julie	Colton	Aidan
		

Colton's roll of 15 gives him a total of 32. Colton is now in the lead, and Aidan must now roll the dice:

Julie	Colton	Aidan
		

Aidan's roll of 17 puts him over a total of 35. Colton is the winner of the game!

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →	Julie	Colton	Aidan
ROUND 1	10	9	12
ROUND 2	11	8	14
TOTAL SCORE	21	17	26
ROUND 3	6	15	17
FINAL SCORE	27	32	~~43~~

Robert Lochel, Hatboro-Horsham SD

THE 35 GAME – CONDUCTING THE LESSON

PART 1: INTRODUCE STUDENTS TO THE GAME AND ITS RULES (10 minutes)

To begin, students will experience the game and understand the rules. Select three student volunteers and conduct the game in front of the class, keeping score with the Student Scoreboard projected onto the board, or using a similar hand-drawn scoreboard. Ensure that all students understand the rules and the goal – to get as close to 35 without going over in 3 rolls.

Arrange students into teams of 3. Provide each team three dice and a student scoreboard. Students play the game in their group and establish a winner. The winners from each group game can then be used in the next part of the lesson to establish a "Class Champion".

If the number of students in the class is not a multiple of 3, some students can be chosen to act as scorekeepers for small-group games.

PART 2: LEVERAGING THE GAME FOR INEQUALITY DISCUSSIONS (20 minutes)

Choose three students to play the game in front of the class – choosing winners from the small-group rounds works nicely. Also, distribute the number line and scoresheet worksheets to each student. Play the first two rounds, having students call out their sums and computing the 2-round total. The Julie – Colton – Aidan game will serve as an example here:

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →	Julie	Colton	Aidan
ROUND 1	10	9	12
ROUND 2	11	8	14
TOTAL SCORE	21	17	26
ROUND 3			
FINAL SCORE			

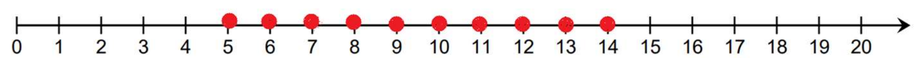
DEVELOP A NUMBER LINE MODEL:

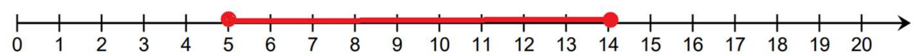
Before Julie's round 3 roll, provide the following verbal prompt:

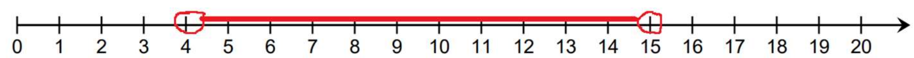
"What are possible rolls for Julie in round 3 which will keep her alive in the game?"

Allow students to discuss this prompt within their small groups, or with shoulder partners. In this game, Julie needs a roll of at least 5 (in order to tie Aidan), but cannot roll a sum higher than 14 (this would put her over 35). As a class, it is helpful to discuss how to deal with ties – generally, ties should be considered acceptable, as co-winners can be awarded.

As students establish a list of acceptable rolls, ask students to use the number line to display the desirable values. Allow student volunteers to share their number lines with the class and compare / discuss the models. Depending on how much experience students have with using a number line to plot values, responses can take on a variety of forms, like the ones shown below:







Additional considerations for the number line discussion:

- Some students may choose closed dots on the number line to display possible rolls (discrete approach), while others may shade all numbers within an interval (continuous approach). A discrete approach is acceptable and correct at the start of the discussion, though discussions will move towards the continuous case as students begin to consider general algebraic inequalities.
- Some students may use open circles to mark acceptable rolls. It is helpful at this point to encourage closed circles to represent acceptable rolls – solutions. Using ordered pairs and their closed circle representations on the plane can help emphasize consistency in communication.

DEVELOP A VERBAL MODEL:

Next, ask students to write a verbal description of acceptable rolls. Have students share their models within their small groups. The teacher can circulate and locate a handful of contrasting verbal models to for whole-class discussion and invite students to share their verbal models in front of the class, or on individual dry-erase boards. Here are a few possible student-generated models, and possible teacher responses:

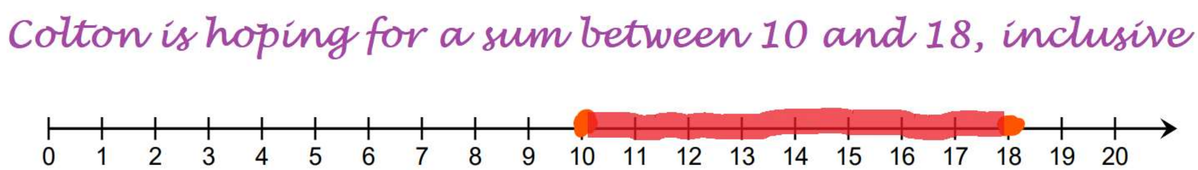
- “Julie needs a roll between 5 and 14.”
    - Ask students if they feel this model clearly expresses the acceptable rolls. In particular - does this model include 5? Does it include 14? This is an ideal time to introduce “inclusive” as a descriptor. This description will eventually be expressed using the algebraic notation $5 \leq x \leq 14$.

- "Julie's roll needs to be greater than 5, but less than 14"
    - There are two items of emphasis here. First, emphasize precision in using "greater than or equal to" within the description. If a student offers "greater than", allow for debate within the class regarding the precision of the statement. Also, guiding students to use "and" as the connector between the 2 inequalities will allow the class to compare the language of conjunctions and disjunctions later ("and" vs "or").

After both the number line and verbal models has been shared, Julie completes her round 3 roll. In the example game, Julie rolls a sum of 6. The current scores are:

Julie 27, Colton 17, Aidan 26

Before Colton competes in round 3, student teams complete both number line and verbal models for Colton's acceptable rolls. This task serves as an additional opportunity reinforce precision in communication, and responses will hopefully demonstrate application of ideas from the previous discussion. The teacher can select student works to share with the class, or recruit student volunteers to provide answers on the board. A sample response appears below.



After both the number line and verbal models has been shared, Colton completes his round 3 roll. In the example game, Colton rolls a sum of 15. The current scores are:

Julie 27, Colton 32, Aidan 26

THE FINAL ROLL – FLIPPING THE SCRIPT

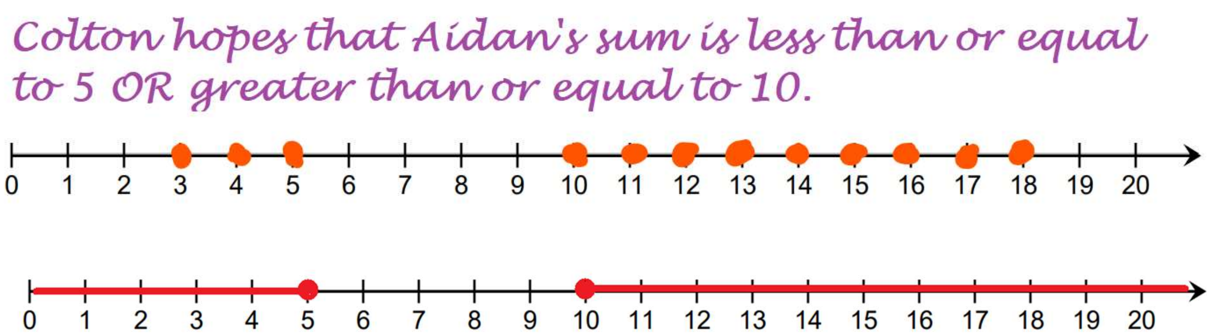
The final player is Aidan. For this final die roll, the prompt asks students to shift their perspective:

"What sums are Colton hoping to see when Aidan rolls, if Colton hopes to win the game?"

This prompt will often lead to a disjunction, rather than a conjunction, as the player in the lead will hope for a sum which is either too low or too high from the current student roller. Ask student teams to develop number line and verbal models for this new prompt. Here are some areas of focus for teachers as student work is discussed:

- Continue to emphasize precision in verbal description. In particular, students should use "greater than or equal to" and "less than or equal to" appropriately.
- Use this opportunity to compare the meanings of "and" and "or" in verbal descriptions.
- Since this dice-based game is played on a domain of 3 through 18, inclusive, student number line models will often stop at 3 and 18. This is acceptable and correct for the game, and a transition to the infinite number line should occur as the dice game context is removed and the class moves to general algebraic models.

Sample student responses:



Colton then completes his roll for the final round, and the game winner is determined!

Classroom Tip: if the game played in front of the class does not provide opportunity to discuss strategy or bring in compound inequalities, simply award the winner and start a new game.
Probability is a wonderful, yet fickle, entity!

PART 3: A MOVE TOWARDS FORMAL SYMBOLIC REPRESENTATIONS (20 minutes)

In the next segment of the lesson, the teacher will introduce algebraic models which represent possible sums. This can be done on the same day the game is introduced, or the next day as a continuation of the lesson. Consider this game, where the first two rounds are complete:

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →	Allison	Carolyn	Chase
ROUND 1	12	9	11
ROUND 2	7	13	14
TOTAL SCORE	19	22	25
ROUND 3			
FINAL SCORE			

The class now considers possible sums for Allison and develops a verbal model.

"Allison needs a roll greater than or equal to 6, and less than or equal to 16." *OR*

"Allison needs a roll between 6 and 16, inclusive."

If the class only provides one of the possibilities shown above, the teacher can then provide the symbolic model, followed by a challenge to the class to consider another way a verbal model could be constructed. In all models, the unknown (x) will be used to represent the sum for the next player.

"Allison needs a roll greater than or equal to 6, and less than or equal to 16."

$$x \geq 6 \text{ and } x \leq 16$$

"Allison needs a roll between 6 and 16, inclusive."

$$6 \leq x \leq 16$$

This presents a wonderful opportunity to compare the symbolic models and debate their efficiency. Allow students to choose a model which makes sense to them yet understand and be able to explain the structure of each expression. Allison then rolls the dice:

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →	Allison	Carolyn	Chase
ROUND 1	12	9	11
ROUND 2	7	13	14
TOTAL SCORE	19	22	25
ROUND 3	8		
FINAL SCORE	27		

By now students can hopefully move between the three types of models (number line, verbal and symbolic) and can express Carolyn's possible sums:

"Carolyn needs a roll greater than or equal to 5, and less than or equal to 13."

$$x \geq 5 \text{ and } x \leq 13$$

"Carolyn needs a roll between 5 and 13, inclusive."

$$5 \leq x \leq 13$$

After Carolyn complete her round 2 roll (here, she has gone over 35), the class will now consider the sums Allison hopes to see from Chase in his final roll.

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →	Allison	Carolyn	Chase
ROUND 1	12	9	11
ROUND 2	7	13	14
TOTAL SCORE	19	22	25
ROUND 3	8	16	
FINAL SCORE	27	~~38~~	

In this situation, note that there is no roll "too low" for Chase. He can only lose the game if he rolls a sum higher than 10, but there is still more than one verbal way Allison can express her hopes, and they are worth comparing, along with their symbolic models.

"Allison wants Chase to get a sum greater than or equal to 11." $x \geq 11$

"Allison wants Chase to get a sum greater than 10." $x > 10$

In many games the final prompt will lead to a disjunction, which can serve as a comparison to the conjunctions observed with the first 2 players. For example:

"Freddy wants Julia to get a sum less than or equal to 6 OR greater than or equal to 16."

$x \leq 6$ or $x \geq 16$

PART 4: FORMAL PRACTICE WITH INEQUALTIES

From here, students should have a foundation with the three types of models used to express inequalities: number lines, verbal models, symbolic models. Traditional student practice should offer opportunities for fluency between the three models.

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →			
ROUND 1			
ROUND 2			
TOTAL SCORE			
ROUND 3			
FINAL SCORE			

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →			
ROUND 1			
ROUND 2			
TOTAL SCORE			
ROUND 3			
FINAL SCORE			

THE 35 GAME – STUDENT SCOREBOARD

PLAYERS →			
ROUND 1			
ROUND 2			
TOTAL SCORE			
ROUND 3			
FINAL SCORE			

THE 35 GAME – STUDENT NUMBER LINES